Preprint
UCRL-JC-137887

An Experimental Investigation of a Prescription for Identifying Plastic Strain

A.A. Brown, J. Casey, D.J. Nikkel, Jr.

This article was submitted to
8th International Symposium on Plasticity and its Current Applications, Whistler, British Columbia, Canada, July 17-21, 2000

February 29, 2000

U.S. Department of Energy

Lawrence
Livermore
National
Laboratory

AN EXPERIMENTAL INVESTIGATION OF A PRESCRIPTION FOR IDENTIFYING PLASTIC STRAIN

Arthur A. Brown*, James Casey*, and Daniel J. Nikkel, Jr.**

***Department of Mechanical Engineering**
University of California at Berkeley, Berkeley, CA 94720

****New Technologies Engineering Division**
Lawrence Livermore National Laboratory, Livermore, CA 94551

ABSTRACT: A series of experiments is described in which a novel prescription for the identification of plastic strain is tested to determine its validity in the context of the strain-space formulation of rate-independent plasticity. Biaxial experiments were performed on several thin-walled aluminum 1100-O cylindrical specimens.

INTRODUCTION: Although plastic strain is a key element in most plasticity theories, controversy still exists on how to identify it in the context of finite deformations. Numerous methods for measuring plastic strain have been suggested in the literature, but most are only valid for certain types of material behavior.[1] Casey and Naghdi [1992] proposed a prescription for identifying plastic strain that can be used for both finite and infinitesimal deformations.

THEORETICAL BACKGROUND: Let the tensors $\mathbf{E}$ and $\mathbf{S}$ denote the Lagrangian strain and the symmetric Piola-Kirchhoff stress. The plastic variables $\mathbf{E}^p$, κ, and α represent the plastic strain, the work hardening parameter, and the shift tensor. There exist a stress response, $\mathbf{S} = \hat{\mathbf{S}}(\mathbf{E}, \mathbf{E}^p, \kappa, \alpha)$, its inverse, $\mathbf{E} = \hat{\mathbf{E}}(\mathbf{S}, \mathbf{E}^p, \kappa, \alpha)$, and a smooth yield function $g(\mathbf{E}, \mathbf{E}^p, \kappa, \alpha)$ in strain space, such that $g = 0$ represents a smooth yield surface that encloses the elastic region in strain space. For unloading or neutral loading, the plastic variables remain fixed. During loading, $\dot{\mathbf{E}}^p = \rho\hat{g}$, $\dot{\kappa} = \lambda\hat{g}$, and $\dot{\alpha} = \beta\hat{g}$, where ρ, λ, and β are constitutive response functions that depend on $\mathbf{E}$, $\mathbf{E}^p$, κ, and α.

The prescription for identifying plastic strain is stated as follows: (1) Determine the point on or within the yield surface in stress space that minimizes the value of the norm $\|\mathbf{S}\| = (\mathbf{S}\cdot\mathbf{S})^{1/2}$ of the stress tensor. Let $\mathbf{S}^p$ denote the stress at that point. (2) Unload by any elastic path to the stress $\mathbf{S}^p$. (3) Measure the strain corresponding to $\mathbf{S}^p$. This value of strain is identified as the plastic strain, $\mathbf{E}^p$.

EXPERIMENTAL SETUP: Extruded aluminum 1100 tubing was machined into 9 in. long cylinders having dimensions of 2.017±0.001 in. inside diameter and 0.100±0.001 in.

[1] For example, identifying plastic strain as the strain corresponding to zero stress is unacceptable in cases where the yield surface in stress space does not enclose the origin.

wall thickness. The cylinders were annealed at 650°F for one hour, resulting in an average grain size of 0.003 in. Tensile, compressive, and torsional loads were applied to the specimens by an MTS closed-loop hydraulic testing machine. Strains were measured using 45° rosettes. A 5×10^{-6} strain offset definition of yield was adopted. Load increments of 85 psi were applied at a rate of one increment per minute. During plastic loading to a new yield surface, strain rates were kept under 20×10^{-6}/min.

PROCEDURES, RESULTS AND DISCUSSION: Results from one of the aluminum specimens will be reported here. Three tests were conducted, each using the following procedure (refer to Figs. 1 - 3): First the specimen was preloaded to an elastic-plastic State A, and the corresponding yield surfaces in stress and strain space were determined. The specimen was then unloaded to $\mathbf{S}^p_A$, and $\mathbf{E}^p_A$ was measured. Next, the specimen was loaded to State B by applying a strain increment $d\mathbf{E}_1$ and corresponding stress increment $d\mathbf{S}_1$. The yield surfaces for State B were determined, and the new plastic strain $\mathbf{E}^p_B$ was measured after unloading to $\mathbf{S}^p_B$. Starting from the same preload point used for State B, the specimen was loaded to State C by applying a strain increment $d\mathbf{E}_2$ and corresponding stress increment $d\mathbf{S}_2$. Maximal unloading was performed to $\mathbf{S}^p_C$, and $\mathbf{E}^p_C$ was measured.[2]

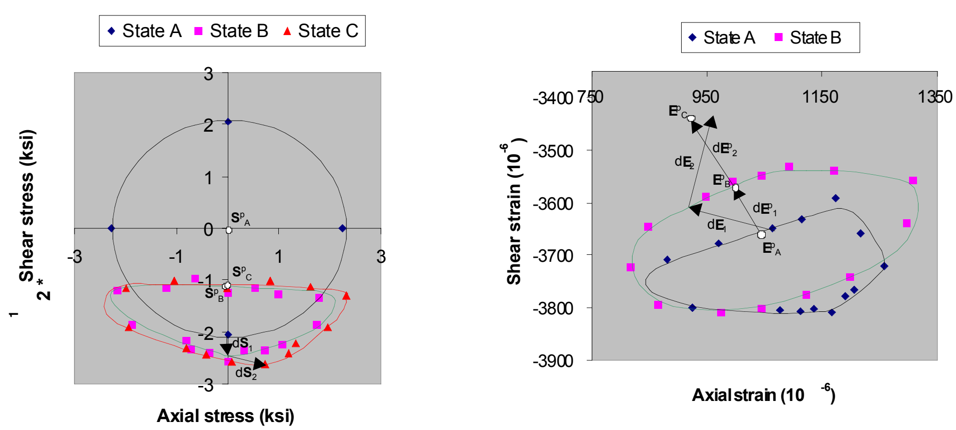

Figure 1: Test 1 in (a) Stress space, and (b) Strain space

The experiments were designed to test if the plastic strain found using the above prescription would satisfy a flow rule of the form $\dot{\mathbf{E}}^p = \rho \hat{g}$. The fact that ρ depends only on the current state implies that the direction of the plastic strain rate is independent of the direction of the strain rate during loading. Hence, the plastic strain increments $d\mathbf{E}^p_1$ and $d\mathbf{E}^p_2$ resulting from the two strain increments $d\mathbf{E}_1$ and $d\mathbf{E}_2$ should have

[2] For Tests 2 and 3, it was not necessary to find the yield surfaces for State C; the origin in stress space was found lie within the elastic region, so $\mathbf{S}^p_C = \mathbf{0}$, and hence $\mathbf{E}^p_C$ was the strain at zero stress.

approximately the same direction in strain space.[3] In Test 1, the plastic strain increments do not appear to be parallel (possibly due to the high curvature of the yield surface at the point of loading). However, in Tests 2 and 3, $d\mathbf{E}_1^p$ and $d\mathbf{E}_2^p$ are approximately parallel in strain space even though $d\mathbf{E}_1$ and $d\mathbf{E}_2$ have radically different directions. The results are in agreement with the flow rule postulated in the strain-space formulation.

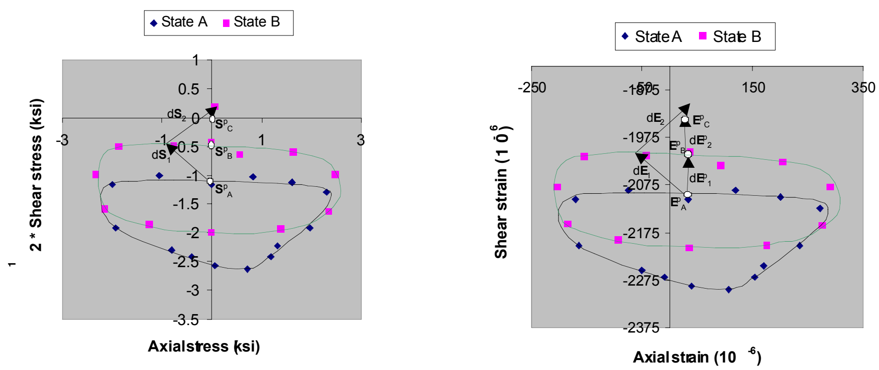

Figure 2: Test 2 in (a) Stress space, and (b) Strain space

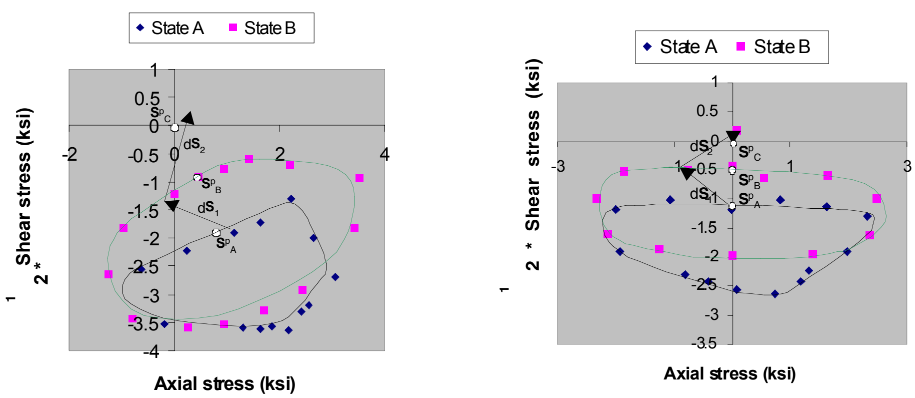

Figure 3:

ACKNOWLEDGEMENTS: The work reported here was supported by the National Science Foundation under Grant No. MSS-9216069, to the University of California. This work was also performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

REFERENCE:

Casey, J. and Naghdi, P. M. 1992, "On the identification of plastic strain at deformation", in Defects and Anelasticity in the Characterization of Crystalline Solids, Brock, L. M. (ed), ASME AMD Vol. **148**, 11

---

[3] The assumption implicit in this argument is that the direction of the response function ρ in the flow rule remains approximately constant along the incremental paths $d\mathbf{E}_1$ and $d\mathbf{E}_2$.